Student’s Name: Ayush Kapoor

Instructor’s Name: Matt Hudelson

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Application of Graph Coloring

Graph coloring, also referred to as vertex coloring, is a process of coloring graph vertices so that two adjacent vertices do not have matching colors. Graph coloring, which is a special case of graph labeling, is more emphasized by vertex coloring because it is used to address most of its coloring problems, and also since other forms of coloring can be transformed or changed to conform to it. For example, an edge coloring, which assigns colors to edges so that two adjacent edges do not have the same color, can be looked at as vertex coloring of its line graph (Dey 373). In contrast, plane face coloring, which assigns a color to faces or planes such that two adjacent planes have no matching colors, can be interpreted as the vertex coloring of its dual (Czap & Július 72). Graph coloring has been applied in multiple areas, including in planar graphs. The objective of this paper is to explore the significance of Graph coloring in various fields of organization and the overview is presented.

The first application of graph coloring mainly dealt with planar graphs. Its motivation, however, came from Francis Guthrie, who postulated the four-color conjecture. Guthrie wanted to color all of England’s counties in a map, sufficiently using four colors and ensuring that each of the colors was different in neighboring regions (Brusco et al. 23). His brother helped pass this question to his university teacher, who later expressed the problem to William Hamilton back in 1852. The problem got widely recognized when it was raised in the London Mathematical Society in 1879. In that same year, Alfred Kempe claimed to have found a solution to the problem, but Heawood, in 1890, disapproved the theory and went ahead to prove the five-color theorem using the Kempe Idea. The four-color theorem remained unsolved for the rest of that century.

However, in the ensuing century, more research was conducted to reduce the colors to four. Kenneth Appel and Wolfgang Haken finally proved the theorem in 1976. In the theorem, a variety of concepts are used, including several notations. For instance, given a graph labeled G, which contains a pair of sets (V, E) and those sets satisfy a particular property, including V (commonly referred to as the vertex set) must have a finite set. In contrast, E, widely referred to as the edge sets, must record which pair of the vertices are adjacent to each other (Steffen et al. 19).

Formally, there is a requirement that the E elements should be ordered pairs of the vertices (Taklimi & Alinaghipour 27). This implies that V\*V. this implies that E is asymmetric, which leads to the conclusion that if (u, v) є E, then (v, u) є E too. Therefore, every edge in the graph can be taken as an unordered vertices pair. In graph coloring, we deal with graphs that cannot be drawn directly but can be clearly understood since their edges and vertices are defined in a concrete way (Hong & Tokuyama 27). The Kneser graphs is an important class of these graph types.

One of the main central concepts of graph theory is proper coloring given a graph with sets (V, E) with C as the denoted positive integer. Whereby c is considered a proper c-coloring and is a function ϕ: V→[c] whereby if u ̴v then ϕ(u)≠ ϕ(v). The numbers 1,2…c is thought to be different colors, and ϕ assigns a color to each vertex while observing the condition that all adjacent vertices have to ensure that they are of different colors. A graph G is said to be c-colorable if it has a proper c-coloring. ᵪ (G), is considered the chromatic number of G and is explained as the smallest value of c such that the graph is c-colorable (Wood 18). For example, a proper 3-coloring can be constructed by ensuring that first every vertex with label one can be given color 1, color 2 can also be put in vertices with a value 2 in their labels and the remaining color 3 to the remaining vertices. This is also one prove that ᵪ (KG (5,2)), an example of a Kneser graph, is ≤ 3. However, their pentagon cannot be 2-colored effectively, implying that ᵪ (KG (5,2)) ˃2. It is, however, essential to note that if the graph has a loop, then it cannot be properly c-colored for any value c.

Edge coloring of a graph L would be the assessment of colors to the graph’s edges with a common endpoint have different colors. Let be the chromatic index of L. Vizing’s theory proposed that is either , where is the maximum degree of a vertex in L. Adding to that, the graph L will belong to either class 1 or class 2. It implied that there are no polynomial-time algorithms for the problem, making it an NP-complete classification problem.

There have been numerous advancements as a result of the diverse research and study of graph theory, especially graph coloring. Its applications have been widely used and have influenced many fields. It has also had a broad base of applications in our daily lives. Since the graph coloring problem assigns colors to a certain element of a subject in a graph, this methodology can be used and interpreted to some other few problems. One of the most issues the problem has solved is the scheduling problem. The vertex coloring model can be used to solve several scheduling problems. For example, in a job scenario, different types of jobs might need to be assigned to a time slot whereby these jobs each need a one-time slot (Zelenova et al. 192). The jobs can be planned in any way, although a conflict may arise if some jobs are not in the same slot even though they depend on a shared resource.

In this scenario, a graph can be drawn that contains a vertex and an edge. The vertex will represent every job. On the other hand, the edge will have the conflicting job pairs. The chromatic number of the graphs will represent the minimum makespan. That is the ideal time needed to complete the jobs avoid of any conflicts.

Taking an example, there are a set of athletes: will all be practicing in different sporting and tracking events for each group such as:

soccer, relay, hurdles}, {tennis, badminton, 400m sprint} {football, basketball, tennis}, {shot put, relay, tennis}, {badminton, 400m, relay}, {hurdles, football, shot put} {basketball, football, soccer}, {relay, soccer, hurdles}

We assume Z as the set of athletes and J = {1 ,2 ,3 ,4 ,5 ,6 ,7 ,8 ,9 ,10} as the set of tracking/sports events that take place with respect to a timeframe. Z(j) will be the set of athletes that will be on the field and construct a graph B = B (J, E) where a, b J will only be adjacent if . Using vertex coloring on the graph B will relent a well-designed field schedule with the vertices in the color class representing the schedule with respect the day. The graph coloring of the field/track scheduling is denoted by figure 1 below.

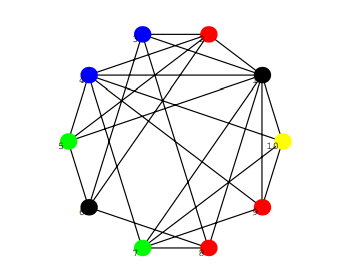


Figure 1

Therefore, using graph coloring, we arrive at the conclusion that only 5 days are needed to execute such a field competition.

Another example of these scenarios can be in assigning aircraft to flights. If two flights overlap, then they cannot be assigned the same aircraft. In this instance, the conflict graph that will ensure the problem is solved efficiently will be the interval graph, which can be colored optimally in polynomial time (Kumar & Pruthi 147.). In the case of the allocation of bandwidth to radio stations, the corresponding graph will be a unit-disk graph. Therefore, the coloring problem will be 3-approximable. One of the very recent technologies on the internet would be webMathematica (based on Java technology), developed by Wolfram research that has provided instructors to compute visual results of complex constructions straight from the web browser.

In conclusion, graph coloring has made some significant progress in new trends. The proof of the 4-coloring problem has greatly impacted the whole coloring theory of both graphs and hypergraphs. The theory has grown to solve numerous current issues in the new century. First, it has been used to ensure that tasks which get the same color can be simultaneously performed since their resources are concurrently available, which has introduced a lot of efficiency in fields associated with resource allocation and optimization. However, there is still a lot that is yet to be discovered in the area. In the many sectors, especially in operating systems and computer science, graph coloring is also making huge advancements that will make considerable contributions in the future (Vinutha & Arathi 15). Bi-Processor tasks on operating systems by task allocation (assignment of colors to the edges of the graph) such that a vertex will appear only once. Data structures can be designed in the form of trees which utilizes vertices and edges. Similarly modelling of network topologies can be implemented through graph concepts. Other fields include image processing, software engineering, website designing, and database designing, which lead to the development of new algorithms and theorems that can be used in tremendous applications in the coming years.

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